

19 Dec. 2024
[This question paper contains 8 printed pages.]

7047
Your Roll No.....

Sr. No. of Question Paper : 1085

I

Unique Paper Code : 2222012301

Name of the Paper : Mathematical Physics – III

Name of the Course : B.Sc. (H) Physics

Semester : III

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. **All** questions carry equal marks.
4. Question number **1** is compulsory.
5. The principal branch of argument of complex number is to be taken as $0 \leq \theta \leq 2\pi$.

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5. Use the following definition for the Fourier transform of $f(x)$:

$$\mathcal{F}[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$

6. Use the following definition for the Fourier sine transform of $f(x)$:

$$\mathcal{F}_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin(kx) dx$$

7. Use the following definition for the Fourier cosine transform of $f(x)$:

$$\mathcal{F}_c[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos(kx) dx$$

8. Use the following definition for the convolution of two functions $f(x)$ and $g(x)$:

$$(f * g)(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(y) g(x - y) dy$$

9. Some useful formulae are given at the end.

1. Attempt any **six** questions :

(6×3=18)

(a) Obtain all the roots of the equation :

$$z^3 - i = 0; i = \sqrt{-1}$$

(b) Show that $\tanh^{-1}(z) = \frac{1}{2} \ln \frac{1+z}{1-z}$

(c) In the finite z -plane, determine and classify the singularities of the function:

$$f(z) = \tan^{-1}(z^2 + 4z + 5)$$

(d) Solve $\frac{1}{2\pi i} \oint_C \frac{e^z dz}{z-1}$; C is $|z-1| = 2$.

(e) Find the residue at $z=0$ for $f(z) = \frac{\cosh(z)}{z^3}$.

(f) If $\mathcal{F}^{-1}[F(k)] = f(x)$, show that

$$\mathcal{F}^{-1}[F(k-a)] = e^{iax} f(x); a > 0.$$

(g) General solution of 1-d wave equation is given as:

$$y(x, t) = \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi x}{L}\right) \cos\left(\frac{n\pi t}{L}\right),$$

If $0 \leq x \leq L$ and $y(x, 0) = x$, determine c_n .

2. (a) If $z = 4e^{i\pi/3}$, evaluate $|e^{iz}|$. (4)

(b) Given $\tan(x + iy) = u + iv$, show that

$$u = \frac{\sin(2x)}{\cos(2x) + \cosh(2y)}$$

and
$$v = \frac{\sinh(2y)}{\cos(2x) + \cosh(2y)} \quad (6)$$

- (c) Prove that the function $u(x, y) = 2x(1 - y)$ is harmonic and hence find $v(x, y)$ such that $f(z) = u + iv$ is analytic. Also, express $f(z)$ in terms of z , where $z = x + iy$. (8)

3. (a) State and verify Cauchy's theorem for the function:

$$f(z) = 3z + 2i$$

and C is a triangle with vertices $1 + i$, $-1 \pm i$. (8)

- (b) Solve the integral

$$\oint_C \frac{z^2 dz}{(z^2 + 9)(z^2 + 4)^2}; \quad C \text{ is } |z| = 1 \quad (5)$$

(c) Expand

$$f(z) = \frac{z}{(z + 1)(z - 2)}$$

in a Laurent series valid for the annular domain $0 < |z - 2| < 3$. (5)

4. Using residue theorem and suitable contour, solve any two real integrals : (2×9=18)

(a) $\int_0^{\infty} \frac{x^2}{x^4 + 1} dx$

(b) $\int_0^{2\pi} \frac{d\theta}{\cos\theta + 2\sin\theta + 3}$

(c) $\int_0^{\infty} \frac{x \sin 2x}{x^2 + 9} dx$

5. (a) Find $\mathcal{F}^{-1} \left(\frac{1}{k^2 - 4k + 29} \right)$. (8)

(b) Show that $\mathcal{F}_c \left(\frac{1}{\sqrt{x}} \right) = \frac{1}{\sqrt{k}}$. (5)

(c) Obtain the function $q(x)$, if $\mathcal{F}_s[q(x)] = e^{-2k}$. (5)

6. (a) Using the method of separation of variables, find the solution of the following partial differential equation :

$$4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$$

$$\text{such that } u(0, y) = 3e^{-y}. \quad (4)$$

- (b) Solve one-dimensional heat equation:

$$\frac{\partial u}{\partial t} = 9 \frac{\partial^2 u}{\partial x^2} \quad (0 \leq x \leq L)$$

$$\text{such that } u(0, t) = 0, u(L, t) = 0$$

$$\text{and } u(x, 0) = x(L - x) \quad (14)$$

Some useful formulae:

$$1. \quad \int_0^\infty x^n \exp(-ax^m) dx = \frac{1}{m a^{(n+1)/m}} \Gamma\left(\frac{n+1}{m}\right);$$

$$n > -1; a, m > 0$$

$$2. \quad \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$3. \quad \mathcal{F}^{-1}\left(\frac{1}{k^2 + a^2}\right) = \frac{\sqrt{2\pi}}{2a} e^{-a|x|}; \quad a > 0$$

$$4. \quad \mathcal{F}^{-1}[a g(k) + b h(k)] = a \mathcal{F}^{-1}[g(k)] + b \mathcal{F}^{-1}[h(k)]$$

(a and b are constants)

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27/12/24

Your Roll No.....

Sr. No. of Question Paper : 1142

I

Unique Paper Code : 2222012302

Name of the Paper : Thermal Physics

Name of the Course/Mode : B.Sc. Hons. (Physics) NEP: UGCF-2022

Semester : III

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt total of five questions out of which Question 1 is compulsory.
3. Use of non-programmable scientific calculator is permissible.
4. Some useful constants are given at the end.

1. Attempt any six (18)

(a) How does Zeroth law of thermodynamics lead to concept of temperature ?

(b) How does the first-order differ from the second-order phase transitions?

(c) Show that $\left(\frac{\partial U}{\partial V}\right)_T = 0$ for an ideal gas.

(d) Explain the need for the second law of thermodynamics when the energy conservation law exists.

(e) Can the change in entropy be a negative quantity? Support your answer.

(f) Calculate the root mean square speed of oxygen molecules at 300K. Take mass of oxygen molecule as 5.31×10^{-26} kg.

(g) Discuss the limitations of van der Waal's equation of state.

(a) What is Adiabatic Lapse Rate ? Derive an expression for the rate of change of temperature with height during the adiabatic gas expansion. How does the presence of water vapour in air affect this process? (6)

(b) One mole of an ideal gas ($\gamma = 1.4$) initially kept at 27°C is adiabatically compressed so that its pressure becomes 10 times its original value. Calculate its temperature after compression and work done on the gas. (6)

(c) Show that the 100% efficiency of a reversible Carnot's heat engine violates laws of thermodynamics. (6)

3. (a) If two Carnot engines A and B are operated in series such that engine A absorbs heat at temperature T_1 and rejects heat to the sink at temperature

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T_2 while engine B absorbs half of the heat rejected by engine A and rejects heat to the sink at temperature T_1 . If the work done in both cases is equal, show that $T_1 = 3T_2 = 2T_1$. (6)

- (b) The heat capacity of a given crystal of mass m at very low temperature varies like T^3 . Calculate the entropy of the crystal as a function of temperature. (6)
- (c) Show that transfer of heat from a body at higher temperature to a body at lower temperature or vice versa leads to increase of entropy of the interacting system. (6)
4. (a) Using Maxwell's equation to prove (any two) : (2*6)

(i) $C_p - C_v = \frac{T\alpha^2 v}{\beta_T}$

(ii) Ratio of isentropic to isochoric pressure coefficients of expansion is $\gamma/(y-1)$

(iii) $T dS = C_p dT - T V \alpha dP$

Where, S stands for entropy, C_v and C_p specific heat at constant volume and pressure, respectively. α is the coefficient of volume expansion and β_T is the isothermal compressibility.

- (b) Use the path-independent nature of thermodynamic potentials to obtain and derive corresponding Maxwell's thermodynamic relations. (6)
5. (a) Show using Clausius-Clapeyron's latent heat equation that substances which expands on melting have their melting point raised by the increasing pressure, whereas those that contract on melting get their melting points lowered by the increase of pressure. (6)
- (b) Draw the pressure-volume isotherms obtained by Andrew's experiments on CO_2 . Discuss the deviations observed from expected ideal gas behavior. (6)
- (c) How can van der Waals's equation of state can explain Andrew's experiment results ? (6)
6. (a) Obtain the coefficient of thermal conductivity (K) of a gas. Discuss the effect of temperature and molecular diameter on ?. (7+5)
- (b) Calculate the mean free path of the molecules of a gas of diameter 3 angstrom at STP. How does the mean free path change if the temperature is reduced to half of its original value and pressure is doubled. (6)

Some useful constants

Boltzmann constant 1.38×10^{-23} J/K, Gas constant 8.31 J/mole-K Avagadro's number 6.02×10^{23} mol⁻¹ or 6.02×10^{26} kmol⁻¹, $1 \text{ amu} = 1.67 \times 10^{-27}$ kg The symbols like U, V, T, p, γ stand for standard notations. (6)

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02/01/2025

Your Roll No.....

Sr. No. of Question Paper : 1342
Unique Paper Code : 2223012001
Name of the Paper : Numerical Analysis (DSE)
Name of the Course : B. Sc. Hons. - (Physics)
(NEP: UGCF-2022)
Semester : III
Duration: : 2 Hours
Maximum Marks : 60

Instructions for Candidates

1. Write your Roll No. on the the top immediately on receipt of the question paper.
2. Question No. 1 is compulsory.
3. Answer any **three** of the remaining **four** questions.

1. Attempt any five of the following

3x5=15

- (a) Give Geometrical Interpretation of Trapezoidal rule.
- (b) Find the eigen values and normalized eigen vectors of the matrix:

$$\begin{pmatrix} 2 & \sqrt{2} \\ \sqrt{2} & 1 \end{pmatrix}$$

- (c) Find the 1-norm, infinity-norm and condition number of the following matrix:

$$\begin{bmatrix} 1 & -2 & 3 \\ 4 & 0 & -1 \\ 2 & 3 & 5 \end{bmatrix}$$

- (d) Using two-point Gaussian Legendre quadrature formula, evaluate the following integral correct to four decimal places:

$$\int_1^5 \frac{1}{x} dx$$

- (e) Define round-off error and show its effect on adding the numbers

$$100+0.49+0.49$$

2. (a) Define a lower and an upper triangular matrix. Solve the system of equations:

$$-3x_1 + 2x_2 - 3x_3 = -12$$

$$6x_1 + 8x_2 + x_3 = 1$$

$$4x_1 + 2x_2 + 7x_3 = 1$$

by obtaining an LU decomposition of the coefficient matrix A for the above system.

(8)

- (b) Solve the following system of linear equations using the Gauss Elimination Method. Show all intermediate steps, including the formation of the augmented matrix, row operations, and back substitution:

$$3x_1 + 2x_2 - x_3 = 5$$

$$2x_1 - 3x_2 + 4x_3 = 7$$

$$x_1 + 4x_2 + 3x_3 = 10$$

(7)

3. (a) Find the Lagrange interpolating polynomial from the following

x	0	1	2	3
Y=f(x)	10	1	0	1

Hence, find an approximate value of f(2.2)

(7)

- (b) Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial:

x	-7	-5	-4	-1
y	10	5	2	10

Find the approximation of y at x=-3.

(8)

4. (a) Approximate the value of $\int_0^1 e^{-x} dx$ using the Trapezoidal rule and verify the theoretical error bound holds for the same.

(8)

- (b) Approximate the value of integral

$$\int_0^1 \frac{1}{1+x^2} dx$$

Using the Simpson's 3/8 rule.

(7)

5. (a) Given $dy/dx = -xy^2$, $y(0)=2$. Compute $y(0.2)$ using Modified Euler method and take $h=0.1$.

(8)

- (b) Given $dy/dx = 1-2xy^2$, $y(0)=1$. Compute $y(0.1)$, $y(0.2)$ using Runge-Kutta Second order method. (take $h=0.1$)

(7)